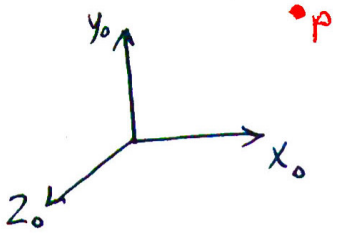
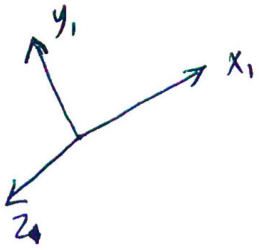
Robotics

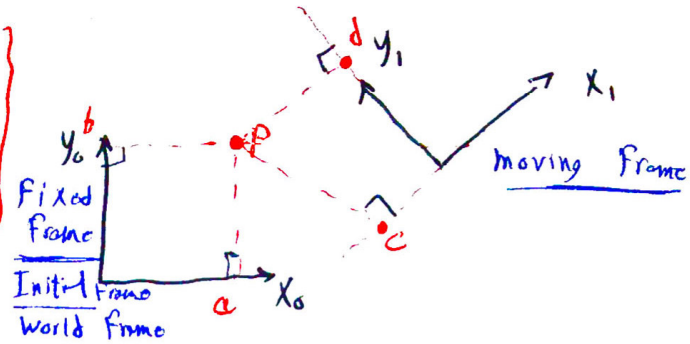
Section 3

Regide motion

- Position Representation
  - ↳ reference frame







$$P = \begin{bmatrix} x \\ y \end{bmatrix}$$

For x_0, y_0

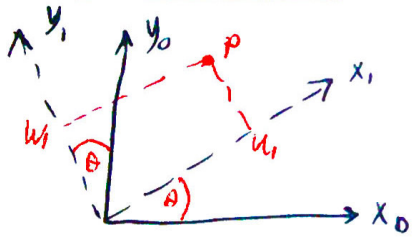
$${}^0P = \begin{bmatrix} a \\ b \end{bmatrix} \simeq \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

For x_1, y_1

$${}^1P = \begin{bmatrix} c \\ d \end{bmatrix} \simeq \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

- for many applications we can Locate an object to the moving Frame
- we need to transform Location from the moving Frame to reference Frame

*Rotation representation



$${}^1P = u_1 \bar{x}_1 + w_1 \bar{y}_1$$

$$\left.\begin{aligned} \bar{x}_1 &= \cos\theta\ \bar{x}_0 + \sin\theta\ \bar{y}_0 \\ \bar{y}_1 &= -\sin\theta\ \bar{x}_0 + \cos\theta\ \bar{y}_0 \end{aligned}\right\} \rightarrow \text{to matrix}$$

$$\begin{bmatrix} \bar{x}_1 \\ \bar{y}_1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \bar{x}_0 \\ \bar{y}_0 \end{bmatrix}$$

$${}^0R_1 = R$$

$$\begin{bmatrix} \bar{x}_0 \\ \bar{y}_0 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{y}_1 \end{bmatrix}$$

$${}^1R_0 = R^{-1}$$

$${}^0R_1 = ({}^1R_0)^{-1}$$

↳ Transform from 1 to 0

$R \rightarrow$ rotation matrix $\in SO(n)$

$$|R| = \pm 1 \quad \text{+ev}$$

$$R^{-1} = R^T$$

$${}^1P = u_1 \bar{x}_1 + w_1 \bar{y}_1$$

$${}^1P = \begin{bmatrix} u_1 & \bar{x}_1 \\ w_1 & \bar{y}_1 \end{bmatrix}$$

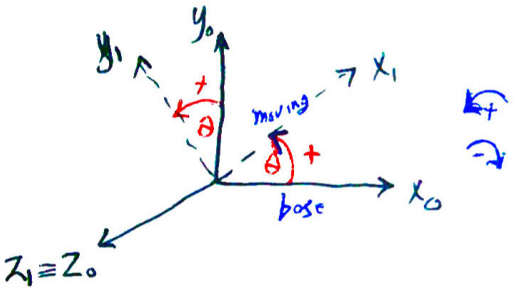
$${}^0P = {}^0R_1 \begin{bmatrix} u_1 & \bar{x}_1 \\ w_1 & \bar{y}_1 \end{bmatrix}$$

$${}^0R_1 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} {}^0x_1 & \vdots & {}^0y_1 \end{bmatrix}$$

$${}^1R_0 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} {}^1x_0 & \vdots & {}^1y_0 \end{bmatrix} = \begin{bmatrix} x_0 \cdot x_1 & x_0 \cdot y_1 \\ y_0 \cdot x_1 & y_0 \cdot y_1 \end{bmatrix}$$

↳ dot product

$${}^{base}R_{moving} = \begin{bmatrix} & & \\ {}^{base}x_{new} & {}^{base}y_{new} & {}^{base}z_{new} \\ & & \end{bmatrix}$$



rotation around z_0

$${}^0R_1 = R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ {}^0x_1 & {}^0y_1 & {}^0z_1 \end{bmatrix} = R(z,\theta)$$

$$R(x,\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \\ {}^0x_1 & {}^0y_1 & {}^0z_1 \end{bmatrix}$$

$$R(y,\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$